MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor A. S. EDDINGTON, F.R.S., PRESIDENT, in the Chair.

Miss Alice G. Bryant, A.B., M.D., F.A.C.S., The Austerfield, 502 Beacon Street, Boston, Mass., U.S.A.;
Charles Ashley Carus-Wilson, M.A., late Professor at McGill University, Montreal, Hanover Lodge, Lansdowne Road, London, W. 11;
Edward Crane, Devonshire House Hotel, Bournemouth;
Henry William Evans, J.P., Harbour House, Solva, Pembrokeshire;
Rev. Louis Alleyan Ewart, Ph.B., F.R.G.S., 13 Twyning Road, Edgbaston, Birmingham;
Luis C. Guérin, Observatorio Nacional, Córdoba, Argentina;
Chester W. Hawkins, Observatorio Nacional, Córdoba, Argentina;
Captain Alva E. Hernandez, Willianmar, Broughton, Hants;
Miss Antonia C. de P. P. Maury, A.B., Hastings on Hudson, New York, U.S.A.;
Captain Arthur Routley Hutson Morrell, Elder Brother of Trinity House, 60 Russell Square, London, W.C.;
John A. Mowney, Melbourne Observatory, Victoria, Australia;
Miss Cecilia Helena Payne, Newnham College, Cambridge, and 70 Lansdowne Road, London, W. 11;
Gordon Peace, Chief Chemist to the British India Corporation, Ltd., The Dale, Cawnpore, India;
Rikichi Sekiguchi, Assistant Director, Imperial Marine Observatory, Kōbe, Japan;
William George Steddy, 64 Northgate Street, Canterbury;
Miss Lorna Mary Swain, Director of Studies and Lecturer in Mathematics, Newnham College, Cambridge;
Metford Watkins, B.A., Director of Mathematics, Fitzwilliam Hall, Cambridge, 86 Clifton Street, Swindon, Wilts;

Robert Winter, Observatorio Nacional, Córdoba, Argentina; and
M. L. Zimmer, Observatorio Nacional, Córdoba, Argentina,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Dinsmore Alter, Professor of Physics and Astronomy, University of Kansas, Lawrence, Kansas, U.S.A. (proposed by H. N. Russell);
A. Frederick Collins, 6 and 7 Tavistock Place, London, W.C. 1 (proposed by P. Y. Alexander);
Howard H. Cox, 38 Braemar Avenue, Wood Green, London, N. 22 (proposed by W. R. Stokes);
Philip Dilk Gilbert, B.A., Weymouth College, Weymouth, and Stratton, Swindon, Wilts (proposed by T. E. R. Phillips);
George Lawrence Harrison, 88 Boulevarde, Hull (proposed by C. H. Adamson);
L. C. Martin, D.Sc., Lecturer on Optics at the Imperial College of Science and Technology, South Kensington, London, S.W. 7 (proposed by A. E. Conrady);
A. M. Mosharrafa, B.Sc., The Manor House, Alphington, Exeter (proposed by W. E. Curtis);
Lt.-Com. Harold Leo Quick, R.A.N., Navy Office, Melbourne, Australia (proposed by A. D. Ross); and
John Shearer, B.A., B.Sc., Observer, Carnegie Institution's Magnetic Observatory, Watheroo, Western Australia (proposed by A. D. Ross).

Fifty-four presents were announced as having been received since the last meeting, including, amongst others:—

San Fernando Observatory, Astrographic Chart, zones $-5°$, $-7°$ (10 charts).
Observatoire Royal de Belgique, Astrographic Chart, zones $+33°$, $+35°$, $+37°$, $+39°$ (30 charts).
International Geodetic and Geophysical Union. First General Assembly, Rome, May 1922. Section of Geodesy. (Presented by Mrs. Grove-Hills.)

Orbits of the Epsilon Arietis Meteors. By C. P. Olivier.

In the *Monthly Notices* for March 1922, p. 309, appears an important and interesting article by Miss A. Grace Cook and J. P. M. Prentice, relative to radiants near the star ϵ Arietis. According to their observations, all made in 1921, these meteors were observed from July 10 to December 31; in this interval 25 separate radiants being derived. The latter were based in six cases upon observations of a single night, in five cases upon a doubly observed meteor, and in fourteen cases upon observations of not more than three and generally two consecutive nights only. The extreme range in right ascension as given for the 25 radiants is only 5°, and in declination only 3°. It is therefore evident, since the observers have taken precautions to avoid numerous errors made by certain others who have found "stationary radiants," and have shown rare persistency in observing over so long a period, that their work deserves the most careful study. It would indeed appear, if ever there was a case of stationary radiation, that we have here an excellent example, especially as the radiants lie almost on the ecliptic. It is apparent that this research was undertaken to confirm the results published by Mr. W. F. Denning in an article upon the ϵ Arietids which appeared in *The Observatory*, vol. xlii. pp. 246–7. Thirty-eight positions of the radiant are here given, ranging from $\alpha = 38°$ to 46,° $\delta = +16°$ to $+23°$. Fifteen at least were doubly observed positions of the same object. The range in time is from July 19 to January 25, inclusive. This is not the place for a review of this article in full, yet the writer feels forced to quote the following sentence which proves that, as late as 1919, Mr. Denning, the chief advocate of "stationary radiation," believed that such a list of radiants could have one parent comet.—"Apparently there is no cometary orbit with suggestive resemblance to this meteor stream, but it may have been, and almost undoubtedly was, derived from a periodical comet which has long ceased to be visible as such, but has distributed its particles around the orbit, and given rise to the annual meteoric displays that are witnessed." Attention must be called to the fact that no matter what assumptions are made or what methods followed, as many different orbits will result from computations as there are different dates. It is true that those radiants, which were observed within a few days of one another, will generally give orbits quite nearly the same, but further on in this article the practical impossibility of such a "stream" being derived from any one parent comet will be mathematically demonstrated, if we use the word stream in the sense Mr. Denning has in sentence quoted above, where it is supposed to be represented by all thirty-eight radiants.

Returning to the paper by Miss Cook and Mr. Prentice, the writer desires to present some notes under three heads: First, to show that certain other data are needed before a definite study of the published results can be made; Second, to examine the orbits deduced which are based upon the parabolic hypothesis, the position for the radiant being taken at about the average of those published; Third, to give any light that the records of the American Meteor Society can throw upon the observational side of the question.

First, we must find whether the zenith corrections are appreciable, and if so, how great. Here we at once meet an omission in the data given, the hour angle of the radiant being needed to calculate this quantity. This we could find if the hours when first and last meteor were seen or even their average were given. The latitude of place of observation is assumed to be $51\frac{1}{2}°$, the mean position of radiant $\alpha = 42°$, $\delta = +22°$, so far as Table I. is concerned. As we only need approximate figures, an accuracy within about $\pm 1°$ is quite sufficient for L and $L-l$. Table I. was then formed, choosing seven well-distributed cases from those published.*

TABLE I.

1922.	L.	$L-l$.	Δz. $Z=30°$.	Δz. $Z=60°$.
	°	°	°	°
July 10	18	333	0·2	0·4
Aug. 1	40	355	0·2	0·3
Sept. 7	76	31	0·2	0·5
Oct. 5	104	59	0·4	0·7
Nov. 4	133	88	1·0	1·2
Dec. 18	176	131	3·1	5·4
Dec. 31	189	145	3·8	6·6

The columns are, respectively, date, longitude of meteoric apex L, $L-l$, the zenith correction Δz for $z=30°$ and for $z=50°$ respectively. The radiant crosses the meridian at $z=30°$, hence this is the most favourable possible case. As this correction decreases the zenith-distance, the declinations published must all be too great, while the right ascensions will need correction when not on the meridian. The size of the correction in the last case for $z=60°$ shows how important this may become. We conclude that this correction is not greater than the error of observation for the first 60 per cent. of the 25 radiants given, but should be included for all radiants in November and December. One would next desire to know what is the maximum distance a meteor's projected path can be from adopted radiant point and the meteor still be considered to belong to the radiant in question. Unless this distance is small, the greatest errors will often arise in deducing the radiants. Finally, at least for November and December, it would be well to have the observed times, and hence the extreme interval between first and last meteor used to determine each radiant. This would allow us to see whether a mean zenith correction could be safely used for the cases mentioned. Until these data are furnished by the authors, one cannot be sure that the radiants really have as small a range in co-ordinates as appears in the published results.

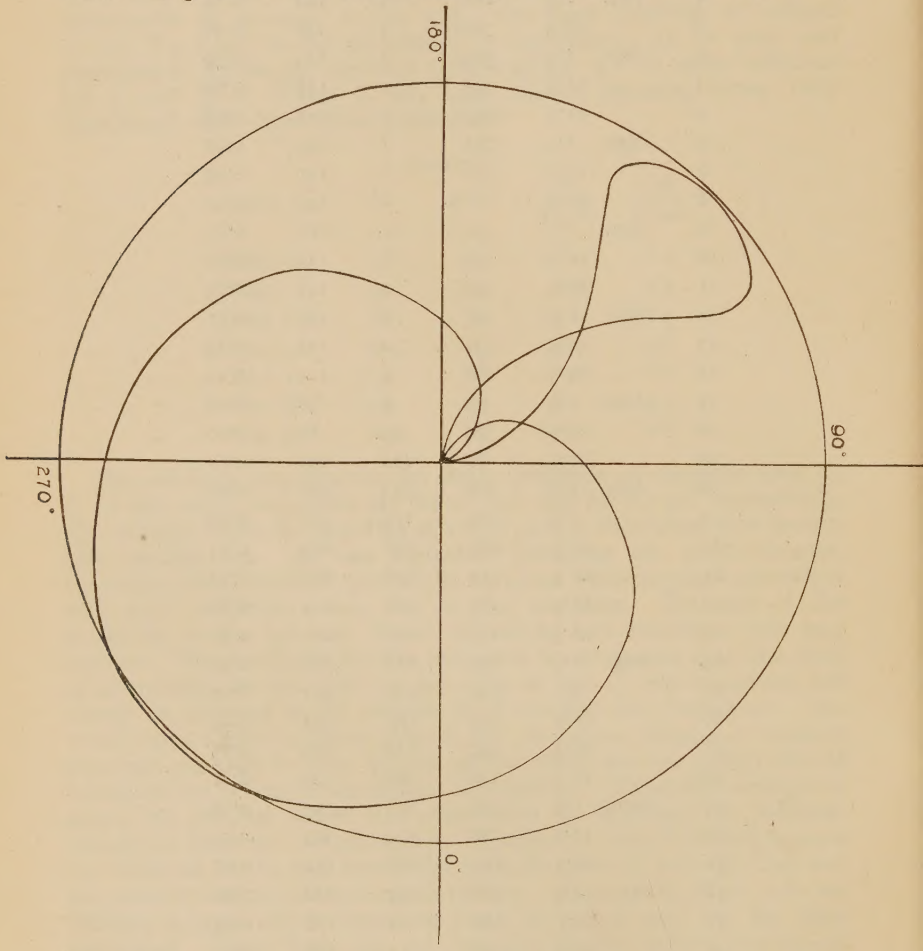
After some preliminary calculations, using published positions, the position $l=45°$, $b=+2°$ was assumed and orbits were calculated. As usual, four-place logarithms were used and 41 parabolic orbits computed. Table II. gives the results, rounded off to the nearest degree.

* The values for Δz in Table I. are taken from *Sternschnuppen*, p. 109, by Schiaparelli.

TABLE II.

No.	Date G.M.T.	Ω.	ι.	π.	q.
	1921.	°	°	°	
1	Jan. 1·5	281	1	122	0·95
2	21·5	301	1	129	0·98
3	Feb. 1·5	313	1	133	0·99
4	11·5	323	1	138	0·99
5	21·5	333	1	141	0·98
6	Mar. 1·5	341	1	144	0·97
7	11·5	351	1	147	0·95
8	21·5	1	1	149	0·92
9	Apr. 1·5	12	1	150	0·87
10	11·5	22	1	148	0·80
11	21·5	31	1	145	0·70
12	May 1·5	41	2	138	0·57
13	11·5	51	2	128	0·39
14	21·5	60	4	115	0·22
15	June 1·5	71	9	98	0·06
16	11·5	80	99	85	0·00
17	21·5	90	170	121	0·07
18	July 1·5	100	174	161	0·26
19	6·5	104	175	181	0·39
20	11·5	109	176	202	0·53
21	16·5	114	176	223	0·67
22	21·5	119	176	244	0·80
23	26·5	123	176	265	0·90
24	Aug. 1·5	129	176	285	0·97
25	6·5	134	177	311	1·01
26	11·5	139	177	332	1·00
27	16·5	144	176	353	0·95
28	21·5	148	176	21	0·81
29	Sept. 1·5	159	176	59	0·59
30	11·5	169	174	103	0·30
31	21·5	178	168	152	0·05
32	Oct. 1·5	188	137	181	0·00
33	11·5	198	9	172	0·05
34	21·5	208	4	156	0·19
35	Nov. 1·5	219	2	141	0·39
36	11·5	229	2	131	0·57
37	21·5	239	1	121	0·73
38	Dec. 1·5	249	1	119	0·81
39	11·5	259	1	119	0·87
40	16·5	265	1	119	0·90
41	21·5	270	1	120	0·91

The figure, based upon the table of orbits, is the locus of the perihelia of the parabolic orbits calculated. The major-axes would hence take as many different positions as there are different values of π. The circle has radius one unit, and therefore nearly represents the earth's orbit. A glance at this remarkable figure will show that only in the



double-looped part could we possibly have the major-axes even approximately parallel, using the latter term in the loosest sense. But during July and August, the figure shows that the probability of any physical connection of meteors, coming from such a fixed point, is absolutely zero. Hence the favourite hypothesis of parallelism of major-axes is untenable. Another striking and unaccountable feature is that the inclination changes from direct to retrograde in June and back again in October, while the perihelia lie practically within the sun itself. The observers found no

radiants for June, but no less than eight for October, including some about the critical date. One of the principal reasons for publishing this figure is to bring to the attention of the observers themselves just what their results, if we accept them as given, would force us to believe to be the structure of the resulting meteoric system. This figure turns out to be not unlike that given by Prof. Plummer in *M.N.*, 81, 138. However, the writer desires to say, at least for himself, that he has not the slightest belief that any such highly artificial and complicated system exists in nature. It may be objected that the parabolic velocity was assumed. But something *has* to be assumed in such calculations, and lacking the true velocities this has long been considered the best practice. Were another velocity assumed, the shape of the curve would certainly be somewhat modified, but there is no reason to expect that it would be any less complicated, or that the above reasoning would not hold equally good.

The writer has carefully searched the published results of the American Meteor Society.* Ten radiants are found somewhere near $\alpha = 42^\circ$, $\delta = +22^\circ$. These are given in Table III.

TABLE III.

Orbit.	Date	G.M.T.		α.	δ.	Meteors.	Remarks.
		m	d	°	°		
42	1909	8	9·88	42	+26	...	
59	1909	8	11·87	44	28	3	
64	1904	8	11·84	42	30	3	
84	1909	10	15·90	50	23	5	
233	1912	8	12·78	46	28	4	
646	1917	8	24·69	40	26	5	Good.
647	1916	10	27·77	40	28	5·7	Good.
670	1917	11	6·60	45	25	5	Good.
672	1916	10	22·69	45	17	9	Good.
676	1916	10	22·74	46	24	6	Poor.

Numbers 42, 59, 64, and 233 obviously belong to a single system at $\alpha = 43^\circ$, $\delta = +28^\circ$, for all are observed within an interval of three days. The other six seem scattered at random. The records of the American Meteor Society include tables of over 1200 radiants, based on about 31,000 meteors observed during 20 years, yet only six other radiants are found within 5° of the assumed fixed point, and but one of these falls within extreme limits of the positions given in paper under discussion. It must be said, however, that no member of the A.M.S. had any particular expectation of finding a radiant near ϵ Arietis. This prevented a special watch in the immediate neighbourhood, but on the contrary there could have been no possible bias. It must therefore be

* *Transactions of the American Philosophical Society*, N.S. 22, Part I. *Publications of the Leander McCormick Observatory*, 2, Parts 4 and 7.

said that the very extensive records of the A.M.S. lend practically no support to the hypothesis of a stationary radiant in the position under consideration.

It is therefore concluded: first, that the authors should furnish more complete data in order that their results can be examined critically; second, that the practical and theoretical reasons against the existence of such a fixed radiant seem almost insuperable; third, that in any case, before their conclusions can be unreservedly accepted this radiant should be independently investigated and proved to exist by other observers.

Leander McCormick Observatory,
University of Virginia:
1922 *October* 25.

Photograph of a Bright Meteor. By Dr. William J. S. Lockyer, M.A.

(Plate 1.)

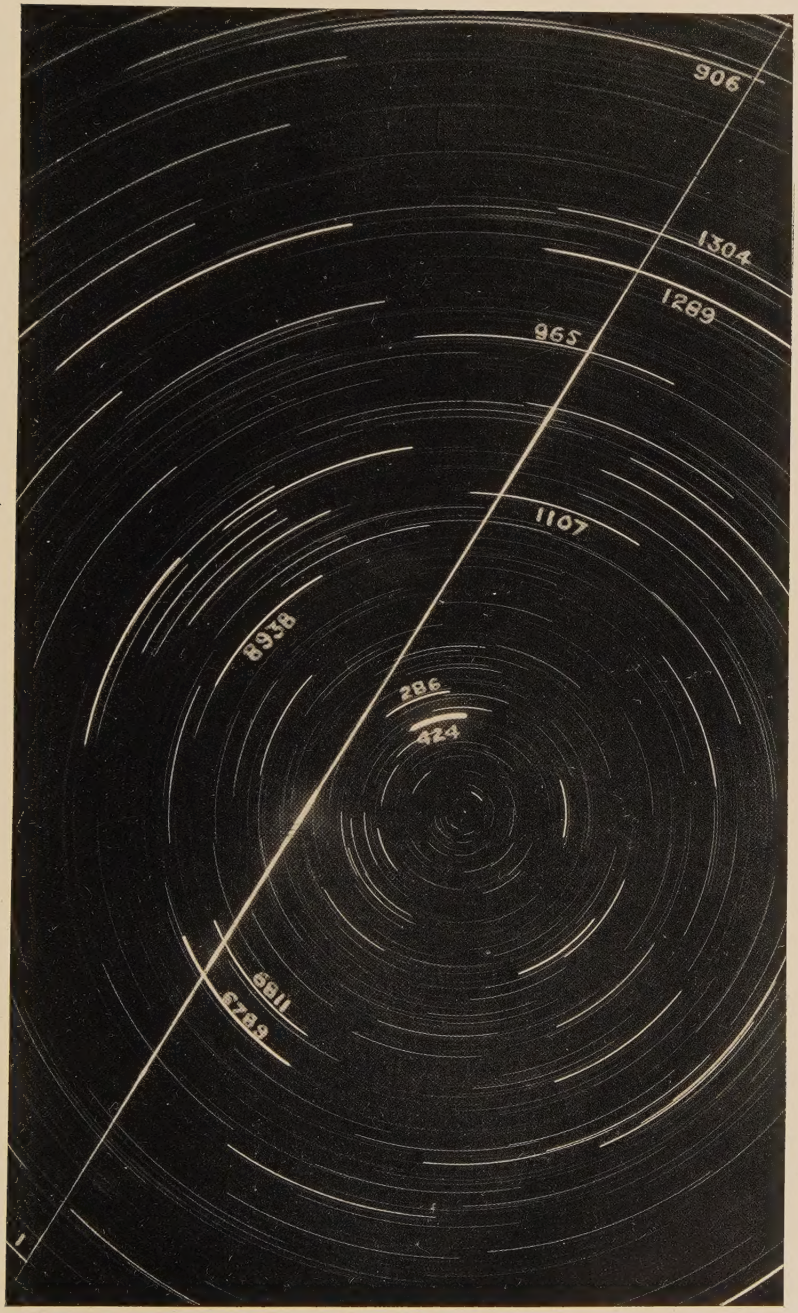
It is my usual practice now at the Norman Lockyer Observatory when working with the 9-inch prismatic camera at night to expose a plate to the north polar region of the sky in a fixed camera outside the dome. This camera is home-made, consisting of a square-section wooden tube with a slot at one end to carry a plate $8\frac{1}{2} \times 6\frac{1}{2}$ inches. The objective is an old portrait doublet of $5\frac{1}{4}$-inch aperture and 28 inches focal length, and includes a rack and pinion for focussing. The whole instrument is mounted in the meridian on a stout wooden frame, the top surface of which is inclined at an angle to suit the latitude.

During the night of November 16 the plate (Marion's "Record" H and D 500) was exposed from $8^h\ 58^m$ to $11^h\ 12^m$ G.M.T., thus covering the period of time of 134 minutes. The weather was cloudless during the whole period of the exposure.

During the development of the plate on the following morning the first images to appear were a long streak across the plate which was the meteor trail, and then the trail of the pole star. According to the *Henry Draper Catalogue* the magnitude of Polaris is 2·12 photometric and 2·62 photographic. After complete development of the plate, star-trails down to the tenth magnitude were recorded.

The plate was "unbacked" and Polaris was sufficiently bright to show very pronounced halation on each side of its trail. The strongest halation was, however, that given by the meteor in that portion of its trail not far from Polaris. Considering the slowness of the movement on the plate of the image of Polaris due to the earth's rotation, and the relatively great speed of the meteor (probably in any portion of its trail only a small fraction of a second), the brilliancy of the latter must have been very great.

Plate 1 gives a reproduction (natural size) from the original negative, but while the halation from Polaris is nearly lost, that from the meteor is still quite apparent. The following table gives the magnitudes of the


906
1304
1289
965
1107
8938
286
424
6811
6783

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stars near the meteor trail taken from the *Harvard Revised Catalogue*, the H.R. numbers being also marked on the plate :—

H.R. 7025	6·15 Mag.	H.R. 1107	5·84 Mag.
6789	4·44	965	5·78
6811	5·86	1289	5·39
8938	5·62	1304	5·70
286	6·48	906	5·95
424	2·12		

The most striking feature of the meteor's trail is the great differences of intensity along the path. Commencing its trail near the star 906 the image is fairly homogeneous until the star 965 is reached, when a sudden increase in brightness is recorded lasting until the star 1107 is approached. A few bright points are seen on the trail before the position of greatest brightness is reached, this occurring about halfway between 424 (Polaris) and 6811. This relatively long outburst was followed by a series of less intense portions of the trail, after which the trail gradually faded away. These outbursts were probably due to the unequal volatilisation of the material forming the meteor, since one must assume that the density of the air in the path of the meteor was fairly homogeneous.

It will be seen from the illustration that while the point of disappearance of the meteor (lower left hand corner of the plate) is shown, the beginning of the trail must have been just off the plate. The trail on the plate is 16° long and quite sufficient to determine the direction of the radiant point. Tracing the path backwards on a celestial globe the trail lies nearly across the middle of the constellation of Taurus, passing between κ and ω Tauri and almost passing over γ Tauri.

According to Mr. W. F. Denning there is a radiant point at κ Tauri (R.A. 60°. Dec. $+23°$) about November 20, which is conspicuous for slow-moving bright meteors. There is little doubt, therefore, that this meteor was a member of this shower, and its intensity on the photographic plate indicates that it was most probably a comparatively slow-moving object.

In order to find out whether this meteor had been observed elsewhere, I communicated with Mr. Denning, and although he kindly made inquiries for me from a number of his correspondents, no one seems to have seen it, probably owing to the very cloudy weather which seems to have been experienced inland.

Mr. Denning makes the following remarks on a direct print from the negative I sent him :—

"I have received your splendid photograph and have been much interested in it. It is the best meteor photograph I have seen, for it seems to include nearly the whole path, and shows the variations in the light and apparent magnitude of the object very distinctly. I have not determined the positions exactly, but from a rough approximation the object appears to have been a Taurid fireball. They are quite abundant at the period of the Leonid shower and until about November 25. . . ."

Mean Areas and Heliographic Latitudes of Sunspots in the Year 1919.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those in the *Monthly Notices*, 82, 483, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at Kodaikánal Observatory and at Dehra Dûn.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun for the year 1919, and Table II. gives the same particulars for each entire year from 1913 to 1919 inclusive. The areas are given in two forms: firstly, projected areas, *i.e.* as seen and measured on the photographs, these being expressed in millionths of the Sun's apparent disc; secondly, areas corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. gives for each rotation in the year 1919 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots. Table IV. gives the same information for each year as a whole from 1913 to 1919. Tables II. and IV. are thus in continuation of similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*; for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii., and for the years 1901 to 1914 on pp. 402 and 403 of vol. lxxvi.

The rotations in Tables I. and III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

The principal features of the record for the year 1919 are as follows:—

1. The slight fall in the mean daily area of Sunspots from 1118 (corrected for foreshortening) for the year 1918 to 1052 for the year 1919. Thus there is a halt after the pronounced fall in activity from the maximum year, 1917, but the decline in the cycle has since continued regularly.

2. The fluctuations in spot activity for the separate rotations are considerable. There is a well-defined maximum in the middle of the year in rotation 879, with a mean daily area slightly higher than that of the most active rotation (860) in the previous year.

3. The mean daily area of faculæ shows the progress of the solar cycle concurrently with the spots, but for the separate rotations the fluctuations in its amount are, as usual, much less than for the spots.

4. Comparing the two hemispheres, the greater activity, as shown by the mean daily area, is continued in the northern. The southern

hemisphere, however, contains the larger number of groups. Of the entire spotted area 53 per cent. occurred in the northern hemisphere.

5. The equatorwards progression of the mean latitude of spots is consistently maintained.

6. On no day in the year 1919 was the Sun's disc free from spots.

7. The number of separate groups observed during 1919 for at least two days was 305, as compared with 404 for 1918. But there were nine large groups of mean area above 700 millionths as compared with five such groups in the previous year. This accounts for the mean daily area being but slightly below that of the year 1918. Of the 305 groups appearing during the year, 138 were in the northern and 167 in the southern hemisphere. Besides these, there were recorded 141 small groups (mostly consisting of one or of two very small spots) seen for one day only; 61 were in the northern and 80 in the southern hemisphere.

Royal Observatory, Greenwich
1922 *November.*

TABLE I.

No. of Rotation.	Rotation Commenced G.M.T. (Civil).		No. of Days on which Sun photographed.	Mean of Daily Areas. Projected. Umbra.	Projected. Whole Spots.	Projected. Faculæ.	Corrected for Foreshortening. Umbra.	Corrected for Foreshortening. Whole Spots.	Corrected for Foreshortening. Faculæ.
	1918–1919	d							
873	Dec.	23·97	27	217	1284	1603	158	918	2004
874	Jan.	20·31	28	282	1736	1447	201	1207	1835
875	Feb.	16·65	27	272	1636	1832	200	1220	2221
876	Mar.	15·97	27	200	1100	1351	141	789	1613
877	Apr.	12·27	28	256	1432	1139	184	1034	1424
878	May	9·51	27	373	2147	1368	262	1540	1699
879	June	5·72	27	455	2507	1582	331	1891	1983
880	July	2·92	27	216	1293	2355	155	968	2730
881	July	30·13	27	380	2055	1355	282	1518	1599
882	Aug.	26·36	28	213	1207	1362	165	956	1601
883	Sept.	22·62	27	227	1273	1356	159	920	1543
884	Oct.	19·91	27	216	1215	1388	153	876	1565
885	Nov.	16·21	28	121	655	1002	89	486	1196
886	Dec.	13·53	27	111	547	1026	75	386	1152

TABLE II.

Year.	No. of Days on which Sun photographed.	Projected. Umbræ.	Projected. Whole Spots.	Projected. Faculæ.	Corrected for Foreshortening. Umbræ.	Corrected for Foreshortening. Whole Spots.	Corrected for Foreshortening. Faculæ.
1913	365	2	10	87	1	7	95
1914	365	40	195	396	31	152	454
1915	365	186	920	1289	137	697	1521
1916	365	186	978	1512	135	724	1785
1917	365	346	2101	1882	247	1537	2305
1918	364	255	1504	1526	188	1118	1882
1919	365	253	1437	1445	182	1052	1729

TABLE III.

No. of Rotation.	Rotation Commenced G.M.T. (Civil).	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
	1918–1919 d						
873	Dec. 23·97	368	11°·38	549	11°·57	−2°·36	11°·49
874	Jan. 20·31	965	10·84	242	9·17	+6·82	10·51
875	Feb. 16·65	1005	10·00	215	13·16	+5·92	10·56
876	Mar. 15·97	372	9·59	417	17·20	−4·56	13·61
877	Apr. 12·27	440	8·04	594	15·43	−5·44	12·29
878	May 9·51	1007	12·65	533	10·18	+4·75	11·80
879	June 5·72	877	11·78	1014	14·13	−2·11	13·04
880	July 2·92	221	8·71	747	10·45	−6·06	10·05
881	July 30·13	549	5·12	970	6·59	−2·36	6·06
882	Aug. 26·36	298	3·40	658	10·27	−6·00	8·13
883	Sept. 22·62	361	6·46	559	12·11	−4·83	9·89
884	Oct. 19·91	604	10·50	272	14·86	+2·61	11·86
885	Nov. 16·21	356	11·89	130	16·73	+4·23	13·19
886	Dec. 13·53	88	6·92	298	9·07	−5·42	8·58

TABLE IV.

Year.	No. of Days on which Sun photographed.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
1913	365	5	24°·81	2·5	20°·08	+9°·81	23°·23
1914	365	99	21·11	53	23·06	+5·71	21·79
1915	365	379	17·95	318	19·75	+0·77	18·77
1916	365	470	15·14	254	17·05	+3·83	15·81
1917	365	860	13·82	677	15·66	+0·84	14·63
1918	364	609	11·97	509	13·69	+0·29	12·75
1919	365	559	9·85	493	11·79	−0·28	10·76

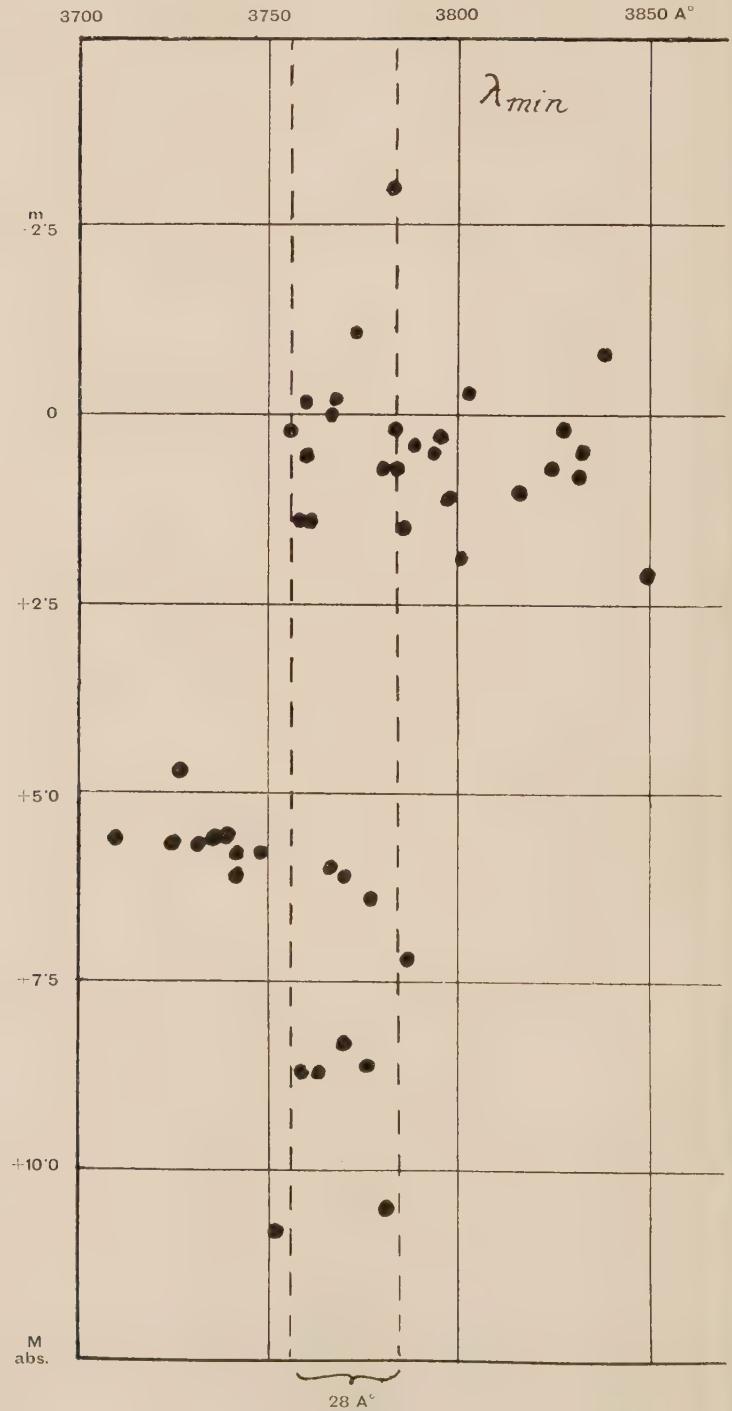

Fig. 1.

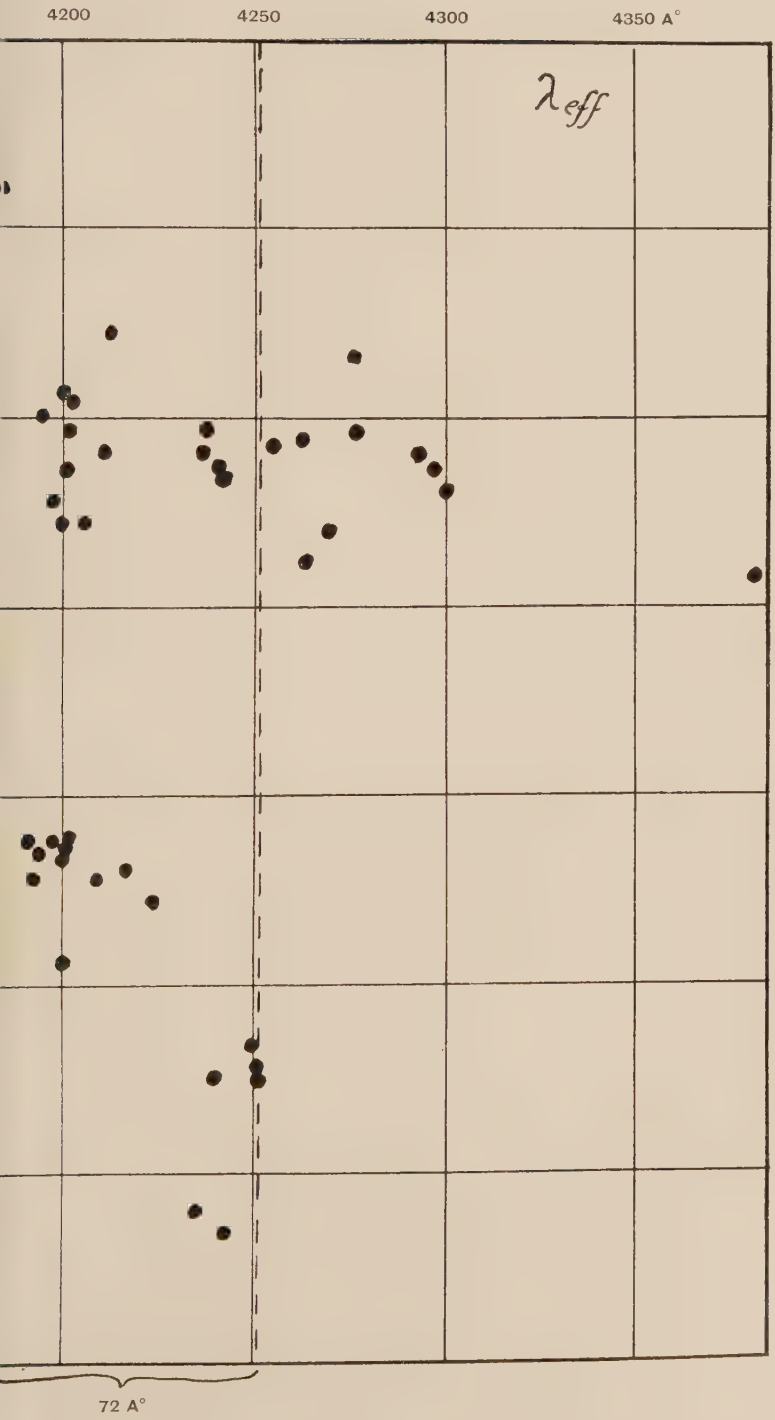

Fig. 2.

Note to Dr. Knut Lundmark's and Dr. W. J. Luyten's Paper on the Determination of the Colour-Equivalent of a Star. By Bertil Lindblad, Ph.D. (Plates 2, 3.)

In a critical paper "On the Determination of the Colour-equivalent of a Star, with Special Reference to the Effective Wave-length, and its Relation to Spectral Class: A Review of the Different Methods," *M.N.*, 82, No. 9, Supplementary Number, Messrs. Lundmark and Luyten have especially treated the effective wave-lengths determined at Greenwich by Davidson and Martin, but also to a certain extent the measurements of λ_{eff} and λ_{min} made by me at Upsala.* Though I appreciate their discussion, which is in many parts very valuable, I feel compelled to emphasise the following points:—

1. It is evident from the diagram (fig. 2) in the above-mentioned paper that a very considerable part of the dispersion in λ_{eff} for the late spectral classes in Davidson and Martin's results may be accounted for by errors of measurement and spectral classification, in addition to the effect of varying luminosity within the types, because of the extreme steepness of the curve between F5 and Ko. Between these spectral classes it seems possible to make a far better estimation of the spectral class from λ_{eff} than of λ_{eff} from the spectral class.

2. The discussion of the combination of λ_{eff} and λ_{min} is misleading. The correlation between these quantities is by no means absolute within the errors of measurement, in spite of the high value of the correlation constant, 0·969, found by Lundmark and Luyten. In forming this coefficient, as well as in figures 3 and 4, the stars of low λ_{eff}, for which it would be futile to try a discrimination between different degrees of luminosity by means of λ_{eff} and λ_{min}, must naturally be excluded. If stars with $\lambda_{eff} < 4180$ Å (nearly corresponding to Go) are omitted, which is obviously permissible, the correlation constant of λ_{eff} and λ_{min} for the 45 remaining stars is 0·756, and the figures 3 and 4 in Lundmark and Luyten's paper, with some change of the scale, take the aspect of figures 1 and 2 of this note.† I have drawn in the diagrams the vertical (broken) lines, to the right of which the giants occur, and those to the left of which the dwarfs lie. The qualitative difference between the two colour-equivalents is quite evident, and is not due to a higher accuracy in the measurement of λ_{eff}. λ_{min} *is obviously relatively more sensitive to a change in luminosity within the same spectral type than* λ_{eff}, quite contrary to the statement of Lundmark and Luyten on p. 507. It is clear that this fact can be used to some extent for a discrimination between giants and dwarfs, as stars of different luminosities will show different characteristic combinations of λ_{eff} and λ_{min}.

It is, moreover, hard to conceive how my measurements of λ_{min} can be at the same time illusory, owing to the influence of imperfect guiding, and "extremely accurate." That a very marked and regular effect of luminosity in λ_{min} as plotted against spectral type exists is sufficiently

* "On the distribution of intensity in the continuous spectra of the sun and the fixed stars, etc.," *Uppsala Universitets Årsskrift*, 1920.

† The absolute magnitudes have been revised according to the new results of Adams and his associates in the list of 1647 parallaxes.

well shown in my paper (*loc. cit.*), fig. 7. The mean accidental errors in the final values of λ_{eff} and λ_{min} were estimated below 10 Å for both wave-lengths.

3. The cause of the qualitative difference between λ_{eff} and λ_{min}, as shown in figures 1 and 2 of this note, has been found to depend on the strong "cyanogen" band at $\lambda 3883$ Å, which is much stronger in absorption for giants than for dwarfs in spectral types G5–K5.* The difference in absorption in the region $\lambda\lambda 3840$–3883 Å may occasionally amount to nearly a magnitude between two stars of the same type, one belonging to the giant and the other to the dwarf branch.

4. For other colour-equivalents from spectral regions essentially unaffected by similarly behaving bands those results of Guthnick and Seares are probably valid, which point to a correspondence in the effect of absolute magnitude for different spectral regions, in the sense that the effect cannot be separated from that of a change in spectral class. I have given some theoretical reasons † for believing the *general colour-effect* of the absolute magnitude to be simply a change of effective temperature, and have found this theory strengthened by the correspondence in magnitude between the effect as shown in Osthoff's visual colours and in λ_{eff}.

5. I do not claim the "minimum wave-length" to be universally accepted as a "colour-equivalent," and this not on account of its strong correlation with λ_{eff} when all types are considered together, but because of the considerable personal equation in the measurements, which is certainly greater than for λ_{eff}. The sensitiveness to changes in the optical system is, moreover, still more marked than for λ_{eff}. But my measures of λ_{min} have been of some significance as a step to a spectrophotometric method (with objective prism spectra) for late-type stars which has been published from the Mount Wilson Observatory, and which (together with a method for A stars) is at present successfully employed at Upsala for the proper motion stars in the Greenwich zone.

Astronomiska Observatorium, Upsala:
1922 *November* 14.

Applications of the Theory of the Stellar Absorption-Coefficient.
By A. S. Eddington, M.A., F.R.S., Plumian Professor.

1. According to the theory developed in a previous paper,‡ the absorption-coefficient k depends on the temperature, density, and average molecular weight, according to the formula

$$mkT^{\frac{7}{2}}/\rho = \text{const.} \quad . \quad . \quad . \quad . \quad (1)$$

This should replace the hypothesis in my earlier papers that k is constant for all stars; and the present paper deals with the astronomical consequences of this change, which affects more particularly the theory of dwarf stars.

* *Mount Wilson Contr.*, No. 228; *Astrophysical Journal*, 55, 85, 1922.
† *Loc. cit.*, *Uppsala Universitets Årsskrift*, pp. 59 ff.
‡ *Monthly Notices*, 73, p. 32.

In order to appreciate the need for this change it is not necessary to assent fully to my theory of the absorption-coefficient. The essential part of the formula (1) can be justified on much wider grounds. The absorption and emission of energy by any portion of the star must balance, and the new formula is arrived at by fixing attention on the *emission* of the radiant energy, the problem of direct calculation of absorption being too difficult at present. We can scarcely doubt that the emission per atom must, *cæteris paribus*, be proportional to the number of free electrons available for the atom to capture or otherwise despoil of energy; hence it will vary as the number of free electrons per unit volume. This number is proportional to ρ/m, subject only to an unimportant correction to allow for a small proportion of the "molecules" being not electrons but atoms. Hence, without any elaborate theory, we see that mk/ρ will be a function of the temperature only.

In my earlier work the possibility of k depending on ρ was not contemplated. This was due to a false analogy with the well-known law of absorption for weak streams of radiation; the stellar absorption occurs under "saturation conditions," and the simple linear law of increase with the number of atoms is inapplicable. Treating k as independent of ρ, the observational evidence from the luminosities of giant stars at different temperatures indicated that it must be nearly independent of the temperature; but if k is proportional to ρ, the *same evidence* indicates that it must be proportional to T^{-3}, since ρT^{-3} is constant for giant stars of the same mass. Thus, without any other retouching of the theory, we arrive at the conclusion that mkT^3/ρ is constant. This directly replaces the original law $k = \text{const.}$; but the exponent of T can be varied slightly —to an extent which does not conflict with the observational evidence— so that the formula (1) is also permissible.

The approximate power of T can also be inferred from general theoretical considerations. The number of electrons brought within range of the atom in given time will be proportional to the electronic velocity, *i.e.* to $T^{\frac{1}{2}}$. If the electron is captured, its energy changes from a positive value accurately proportional to T to a negative value which is at least approximately proportional to T. Thus the emission of energy on capture may be regarded as proportional to T. It seems obvious that a swift-moving electron will be more difficult to capture than a slow one; if the difficulty of capture varies as the xth power of the velocity, the emission will from this cause vary as $T^{-\frac{1}{2}x}$. Finally, to obtain the absorption-coefficient, we must divide the absorption by the whole amount of energy flowing through the material; this is proportional to T^4. The result is that k varies as the $(\frac{1}{2} + 1 - \frac{1}{2}x - 4)$ power of the temperature. Hence

$$mkT^{(5+x)/2}/\rho = \text{const.}$$

Naturally we cannot determine x without assuming a special law of capture or other mode of emission of energy; but x is clearly positive and can scarcely be less than 1. In the theory which I prefer we find $x = 2$. I think that most plausible theories would give a value of x either 1 or 2, and it would be difficult to reconcile the observational evidence with a higher value.

The odd half-power of T in equation (1) is a minor embellishment. All the substantial changes in my results arise from the remainder of the expression $mk\mathrm{T}^3/\rho$, as to which it is difficult to entertain any serious doubt. The point which I wish to make clear is that in the main the recalculations of this paper are demanded by very general and elementary considerations; they do not entirely stand and fall with the particular theory of absorption which is adopted for definiteness. Of course if my view of electron-capture by encounter with the nucleus is not accepted, the constant in formula (1) will be regarded as having been determined from astronomical observation, instead of being calculated as a combination of natural constants.

When the star has reached a density so great as to cause deviations from Boyle's law, the internal temperature does not increase so fast as for a perfect gas. Hence by (1) k begins to increase. The outflow of radiation diminishes for a double reason—first, because the temperature gradient is lower, and, second, because the opacity is higher than for a perfect gas. Only the first effect was recognised in my former calculations; thus the effect of a specified deviation from the perfect-gas laws is greater than I had supposed. The known luminosities of the dwarf stars will be accounted for by smaller deviations from the gas laws; in other words, they are not so near maximum compression. Using the sun as standard, I formerly derived $\rho_0 = 4$ as the density of maximum compression; it will have to be considerably higher according to the new formulæ.

We shall now give the necessary development of the theory of dwarf stars.

2. In any star (giant or dwarf) the following two equations will be satisfied—

$$\frac{d\mathrm{P}}{dr} = -g\rho \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\mathrm{H} = -\frac{\mathrm{c}}{k\rho}\frac{dp_\mathrm{R}}{dr} \quad . \quad . \quad . \quad . \quad (3)$$

the first being the ordinary hydrostatic equation for mechanical equilibrium, and the second the equation of radiative equilibrium. Here p_R is the radiation-pressure, viz.—

$$p_\mathrm{R} = \tfrac{1}{3}a\mathrm{T}^4 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and P is the total pressure $p + p_\mathrm{R}$. H is the net outward flow of heat per sq. cm. per sec., c the velocity of light, and k the coefficient of absorption.* From (2) and (3)

$$d\mathrm{P} = \frac{g}{\mathrm{H}}\frac{\mathrm{c}}{k}dp_\mathrm{R} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If L_r is the heat liberated per second (by sub-atomic energy or by contraction) within a sphere of radius r, and M_r is the mass contained within a sphere of radius r

$$g = \frac{\mathrm{G}\mathrm{M}_r}{r^2} \qquad \mathrm{H} = \frac{\mathrm{L}_r}{4\pi r^2}.$$

* For the origin of these formulæ, see *Astrophysical Journal*, **48**, p. 205.

Hence

$$\frac{g}{H} = 4\pi G \frac{M_r}{L_r}.$$

We set

$$\frac{L_r}{M_r} = \eta \frac{L}{M}$$

where L and M refer to the whole star. η will be unity at the boundary of the star, and will presumably increase towards the centre, since the heat liberated per unit mass is likely to be higher for a central region than for the whole star. Accordingly (5) becomes

$$dP = \frac{4\pi cGM}{L} \cdot \frac{1}{\eta k} dp_R \quad . \quad . \quad . \quad . \quad (6)$$

I have made the approximation of $\eta k = \text{const.}$ We have seen in the first paper (p. 40) that with the new law of variation of k this is likely to be a very close approximation, because k decreases towards the centre of the star, and practically neutralises the increase of η. I therefore write $\eta k = k_0$, where k_0 is a constant. We introduce a constant β given by

$$L = \frac{4\pi cGM}{k_0}(1-\beta) \quad . \quad . \quad . \quad . \quad (7)$$

Then (6) becomes

$$dp_R = (1-\beta)dP$$

so that

$$p_R = (1-\beta)P$$
$$p = \beta P$$

and by (4)

$$P = \tfrac{1}{3}aT^4/(1-\beta) \quad . \quad . \quad . \quad . \quad (8)$$

For a gas deviating from Boyle's law I replace the equation of a perfect gas $p = R\rho T/m$, by

$$p\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right) = \frac{R}{m} T \quad . \quad . \quad . \quad . \quad (9)$$

where ρ_0 is a constant. According to this equation ρ_0 is the limiting value of the density under infinite pressure. This form is suggested by van der Waals' equation, but I shall have to apply it considerably beyond the range for which van der Waals' equation is likely to be accurate. I do not propose to defend it on the grounds of the theory of gases, but because it seems a reasonable interpolation formula between the two extreme states of our stellar material, which obeys the gas-laws at low densities and approaches a maximum density ρ_0 when the pressure is infinitely increased. No doubt the formula is capable of improvement, but it seems suitable for preliminary discussion. If we write

$$\rho' = \rho(1-\rho/\rho_0)^{-1} \quad . \quad . \quad . \quad . \quad (10)$$

(9) becomes

$$\beta P = p = \frac{R}{m}\rho' T \quad . \quad . \quad . \quad . \quad (11)$$

Eliminating T between (8) and (11), we obtain

$$P = \kappa \rho'^{\frac{4}{3}} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where

$$\kappa = \left(\frac{3R^4(1-\beta)}{am^4\beta^4} \right)^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad (13)$$

Hence (2) becomes

$$\frac{1}{\rho} d(\rho'^{\frac{1}{3}}) = -\frac{g}{\kappa} dr .$$

Using (10), the left side can be integrated, giving

$$4\rho'^{\frac{1}{3}} \left(1 + \frac{\rho'}{4\rho_0} \right) = -\frac{1}{\kappa} \int g dr \quad . \quad . \quad . \quad (14)$$

This is the equation determining the distribution of density in a dwarf star.

Suppose that a solution of (14) has been obtained for a particular star. If we multiply all masses by M and all lengths by $M^{\frac{1}{3}}$ the densities are unaltered, and hence the left side is unaltered. But gdr is thereby multiplied in the ratio $M^{\frac{2}{3}}$; hence (14) will still be satisfied if κ is multiplied in the ratio $M^{\frac{2}{3}}$.

It will be necessary to make a separate solution by quadratures for every required value of the mean density; but after that the mass can be varied at will by multiplying κ in the ratio $M^{\frac{2}{3}}$.

I have made a number of such solutions and these are collected in Table I.* In each case I take $\kappa = \frac{1}{3}G = 2{\cdot}22 \cdot 10^{-8}$. The quadratures started at the centre of the star and the first column gives the central density in terms of the "maximum density" ρ_0. The second column gives the radius r at which the star terminates according to the quadratures, the unit being $\rho_0^{-\frac{1}{2}}$ cm. The third column gives the total mass in grammes. The fourth column gives the mean density ρ_m in terms of ρ_0; this is obtained immediately from the mass and radius in columns 2 and 3. If we multiply the mass in the third column by the factor necessary to raise it to the sun's mass $1{\cdot}94 \cdot 10^{33}$ gm., we must multiply the value of κ, $2{\cdot}22 \cdot 10^{-8}$ by the two-thirds power of the same factor. The resulting values of $\log_{10} \kappa$ are given in column 5.

TABLE I.

Solution for Dwarf Stars. ($\kappa = \frac{1}{3}G = 2{\cdot}22 \cdot 10^{-8}$).

ρ_c/ρ_0.	$r\rho_0^{\frac{1}{3}}$.	M.	ρ_m/ρ_0.	$\log \kappa$ (M = ⊙).
0·9	2·294	29·70	0·587	13·5564
0·8	1·800	8·325	0·340	13·9247
0·7	1·717	4·112	0·194	14·1289
0·6	1·790	2·592	0·1078	14·2625
0·5	1·958	1·884	0·0600	14·3549
0·4	2·223	1·483	0·0322	14·4241
0·3	2·616	1·245	0·0166	14·4748
0·0	...	0·884	...	14·5739

The last line refers to the limiting values for a perfect gas.

* Also published in *Zeitschrift f. Physik*, 7, p. 379.

3. Our first problem is to find the value of ρ_0 in gm./cm.^{3} The method adopted is to obtain an independent value of $\log \kappa$ from the sun. Then by comparing columns 4 and 5 we find the value of ρ_m/ρ_0 corresponding. Since ρ_m is known for the sun this determines ρ_0.

By (8) and (11)

$$\frac{\mathrm{R}}{m}\rho' \mathrm{T} = \frac{\frac{1}{3}a\beta}{1-\beta}\mathrm{T}^4.$$

Hence

$$\frac{\mathrm{T}^3}{\rho'} = \frac{3\mathrm{R}(1-\beta)}{am\beta} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

According to the theory of the absorption-coefficient (*loc. cit.*, p. 43, equation (16)), we have

$$\begin{aligned} d + \tfrac{1}{2}\mathrm{N}b &= \frac{\mathrm{H}^2 au}{2\cdot 7\pi \mathrm{R}cb} \cdot \frac{\mathrm{A}}{2\mathrm{N}} \cdot mk\frac{\mathrm{T}^{\frac{7}{2}}}{\rho} \\ &= \frac{3\mathrm{H}^2\mathrm{R}u}{2\cdot 7\pi \mathrm{R}cb} \cdot \frac{\mathrm{A}}{2\mathrm{N}} \cdot \frac{1-\beta}{\beta} \cdot k\mathrm{T}^{\frac{1}{2}}\frac{\rho'}{\rho} \quad . \quad . \quad \text{by (15)} \end{aligned}$$

If we apply this to the case $k = k_0$, the value of $\mathrm{T}^{\frac{1}{2}}\rho'/\rho$ must be chosen for the appropriate part of the star. We indicate this by the suffix 0. Then by (7) we obtain

$$d + \tfrac{1}{2}\mathrm{N}b = \frac{12\mathrm{H}u\mathrm{G}}{2\cdot 7b} \cdot \frac{\mathrm{A}}{2\mathrm{N}} \cdot \frac{\mathrm{M}}{\mathrm{L}} \cdot \left(\mathrm{T}^{\frac{1}{2}}\frac{\rho'}{\rho}\right)_0 \cdot \frac{(1-\beta)^2}{\beta} \quad . \quad . \quad (16)$$

where a slight simplification is made since the physical constant $\mathrm{R} = \mathsf{R}_\mathrm{H}$.

Hence by (13)

$$\kappa = \mathsf{R}^{\frac{1}{3}}\left(\frac{8\cdot 1b}{4\mathrm{H}a^2\mathrm{G}u} \cdot \frac{2\mathrm{N}}{\mathrm{A}}\right)^{\frac{1}{3}} \cdot (d + \tfrac{1}{2}\mathrm{N}b)^{\frac{1}{3}} \cdot \left(\frac{\mathrm{L}}{\mathrm{M}}\right)^{\frac{1}{3}} \cdot \frac{1}{m^{\frac{4}{3}}\beta^{\frac{7}{3}}} \cdot \left(\frac{\rho^2}{\mathrm{T}\rho'^2}\right)_0^{\frac{1}{12}} \quad (17)$$

In the actual calculation $(1-\beta)$ is found from (16), and κ is then calculated directly from (13). But (17) is useful to show how uncertainties in the various data will affect the value of κ when (as usually happens for dwarf stars) β is nearly unity, so that the value of the factor $\beta^{\frac{7}{3}}$ is very little affected by uncertainty of the coefficients in (16).

By far the greatest uncertainty arises from m which appears in the $\frac{4}{3}$ power, whereas the other quantities occur only in the $\frac{1}{6}$ power and T_0 only in the $\frac{1}{12}$ power. It is clear that any reasonable error in the other data would be compensated by a very small adjustment of the value of m. I shall therefore give results corresponding to a number of different values of m.

The formula contains a number of physical constants whose values are accurately known; A/2N can be taken as 1·14 for any element except hydrogen without appreciable error; L/M is accurately known for the sun. There remain the quantities $(d + \frac{1}{2}\mathrm{N}b)$ and $(\mathrm{T}\rho'^2/\rho^2)_0$ to be fixed. The latter is found by a preliminary solution to be 20 million degrees for the sun; a very rough value is sufficient.

For $(d + \frac{1}{2}\mathrm{N}b)$ we may either use a theoretical value or a value based on the observational data of giant stars. From the latter I obtained the value $0{\cdot}87 \cdot 10^{-12}$ cm. (*loc. cit.*, p. 39); but the factor of uncertainty is

3 or 4. The theoretical value depends on whether the term $\frac{1}{2}Nb$ (allowing for change of mass with velocity) ought or ought not to be included. If it is retained ($d + \frac{1}{2}Nb$) must certainly be higher, since for iron $\frac{1}{2}Nb = 3{\cdot}6 \cdot 10^{-12}$ cm. It seems best to hedge, and I shall adopt

$$d + \tfrac{1}{2}Nb = 2{\cdot}0 \cdot 10^{-12} \text{ cm.}$$

This would correspond to a mean atom lighter than iron, or, what is perhaps more likely, to a dependence of mass on velocity intermediate between the two theoretical extremes.

Equation (16) now becomes

$$\frac{(1-\beta)^2}{\beta} = [0{\cdot}3134]\frac{L}{M}\left(T\frac{\rho'^2}{\rho^2}\right)_0^{-\frac{1}{3}} \quad . \quad . \quad . \quad (18)$$

which gives for the sun, $1 - \beta = 0{\cdot}0289$. It may be noted that $1 - \beta$ does not involve the molecular weight.*

By (13) we now obtain the first two columns of Table II. In order to avoid interpolation in Table I., I calculate m from κ instead of *vice versa*. The corresponding values of ρ_m/ρ_0 are taken from Table I. The mean density of the sun is $\rho_m = 1{\cdot}378$ gm./cm.^{3}; hence ρ_0 is found. The last two columns give the corresponding central density and temperature (in millions of degrees) of the sun, the latter being determined by the formula

$$T = m\beta\kappa\rho'^{\frac{1}{3}}/R \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which is easily deduced from (15).

TABLE II.

The Sun ($\rho_m = 1{\cdot}378$).

m.	$\log \kappa$.	ρ_m/ρ_0.	ρ_0.	ρ_c.	$10^{-6}T_c$
1·88	14·5739	0	∞	...	...
2·23	14·4748	·0166	83	25	25·6
2·74	14·3549	·0600	23	11·5	20·6
3·21	14·2625	·1078	12·8	7·7	18·4
4·04	14·1289	·194	7·1	5·0	16·2
5·76	13·9247	·340	4·0	3·2	14·4

It is evident that the important constant ρ_0, representing the maximum density to which stellar material can be compressed, cannot be determined at all closely without an accurate knowledge of m. But the range of values within which ρ_0 might reasonably be expected to lie, say from 7 to 16 gm. per c.c., just corresponds to the range of values of m from 3 to 4 which are also most probable. There is nothing forced about this agreement, so that it seems to be an indication that the theory first developed for giant stars works satisfactorily for the sun when account is taken of the limit of compressibility of the material.

The central temperatures in the last column are all much higher than the central temperature of a giant star at the corresponding stage

* A rough value of m has, however, to be assumed in calculating $(T\rho'^2/\rho^2)_0$.

(about $6 . 10^6$ degrees). Owing to the high temperature we must expect strong ionisation and therefore low molecular weight; the high density inhibits ionisation to some extent but will not prevail against the high temperature if Eggert's thermodynamical formulæ are at all near the truth. For that reason I would select the line $m = 3{\cdot}21$, $\rho_0 = 12{\cdot}8$ as nearest to the actual conditions.

Corresponding results for Sirius are given in Table III., but the observational data are now much less accurate. I adopt for Sirius, visual mag. $= -1{\cdot}58$, parallax $= ''{\cdot}376$, mass $= 2{\cdot}43 \times \odot$, effective temperature $= 11{,}000°$, absolute bolometric mag. $= +0{\cdot}92$. I find $1 - \beta = {\cdot}129$.

TABLE III.

Sirius ($\rho_m = 0{\cdot}455$).

m.	$\log(\kappa/M_\odot^2)$.	ρ_m/ρ_0.	ρ_0.	ρ_c.	$10^{-6}T_c$.
1·93	14·5739	0	∞	...	...
2·30	14·4748	·0166	27·4	8·22	29·6
2·50	14·4241	·0322	14·1	5·64	26·6
2·82	14·3549	0600	7·6	3·80	23·8
3·31	14·2625	·1078	4·22	2·53	21·3
4·17	14·1289	·194	2·40	1·68	18·9

It appears from this table that the molecular weight for Sirius must be somewhat lower than for the sun. Thus if $\rho_0 = 12{\cdot}8$, the molecular weight for Sirius is 2·55 against 3·21 for the sun, corresponding to a loss of 20 electrons instead of 16 for atomic weight 54. It is not improbable that the higher central temperature and lower central density of Sirius would cause appreciably greater ionisation.

4. By reversing the procedure we can obtain the luminosity and hence the effective temperature of a star of any assigned mass and mean density, ρ_0 being now supposed known. The steps are (1) obtain κ by Table I.; (2) obtain $(1-\beta)$ by equation (13); (3) obtain T_c by (19); (4) obtain L by (18)*; (5) calculate the surface area from mass and mean density. The emission of heat per unit area is proportional to the fourth power of the effective temperature.

The following results are calculated with a view to determining the maximum effective temperature reached by a small star as it contracts. The value 12·8 is again taken for ρ_0, and $m = 4$ is assumed for trial.

TABLE IV.

Star of mass $\frac{1}{5} \times \odot$ ($m = 4$, $\rho_0 = 12{\cdot}8$).

ρ_c/ρ_0.	Mean Density.	Eff. Temp.	Central Temp.	$1-\beta$.
0·5	·768	1670°	$8{\cdot}70 . 10^6$	·0058
0·4	·412	1830	$8{\cdot}87 . 10^6$	·0092
0·3	·212	1850	$8{\cdot}57 . 10^6$	·0129

* I assume that $(T^{\frac{1}{2}}\rho'/\rho)_0$ is one-third of the value for the centre, thus allowing for the probable increase of η towards the centre. As already explained, a slight inaccuracy in this factor is not serious.

It would have made the result clearer if I had been able to give the next line $\rho_c/\rho_0 = 0{\cdot}2$; but Table I. does not extend so far. There is, however, no doubt that the effective temperature turns between the last two entries, so that the maximum is about 1850°. But stars of this mass are known to rise to temperature above 3000° and appear as M type stars ; hence the value $m = 4$ assumed above cannot be accepted. The central temperature is much lower than for the sun and Sirius and we should not expect so high an ionisation.

Referring to equation (17), we see that if m is varied, $L^{\frac{1}{3}} m^{-\frac{4}{3}} T_0^{-\frac{1}{12}}$ must be kept constant. We neglect consequential changes in $\beta^{\frac{1}{6}}$, which will be very small when β is so nearly unity. Since T_0 will alter proportionately to m by (19), we have

$$\text{effective temperature} \propto L^{\frac{1}{4}} \propto m^{\frac{11}{18}}.$$

Practically the effective temperature varies as the square of the molecular weight. Since stars a little less massive than $\frac{1}{5}\odot$ seem able to reach about 3000°, we should expect the maximum effective temperature for this star to be about 3500°. This would be obtained by increasing the molecular weight from 4 to 5·4.

Collecting the results for the three stars (Sirius, Sun, and small star), we have

Central temperature	26,	18,	12 * million degrees
Molecular weight	2·55,	3·2,	5·4
Ionisation	20,	16,	9 electrons per atom (weight 54).

The change of ionisation seems to accord with what we should anticipate, unless, perhaps, it is rather too rapid.

So long as the theoretical calculation of the ionisation for given temperature and density is uncertain, we have to be content with this general verification that the results are not improbable. I have considered whether it would be worth while to elaborate Eggert's calculations on this problem ; but I understand that a more rigorous theory will shortly be forthcoming from other sources, and prefer to wait.

The following table, similar to Table IV., shows the course of the effective temperature during the evolution of the sun near the turning point of temperature.

TABLE V.

Star of mass $= \odot$ ($m = 3{\cdot}21$, $\rho_0 = 12{\cdot}8$).

ρ_c/ρ_0.	Mean Density.	Eff. Temp.	Central Temp.	$1-\beta$.
0·6	1·378	5860°	$18{\cdot}4 \cdot 10^6$	·0289
0·5	·798	6300	19·5	·0500
0·4	·412	6600	19·5	·0731
0·3	·212	6460	18·4	·0946

* The value in Table IV. has to be increased proportionately to m.

The assumption that the same value of m applies throughout is justified by the constancy of the central temperature. The interesting point is that a star of mass equal to the sun does not rise above 6600°; in my former investigation the maximum was given as 9000°. Sirius (judging from its mean density 0·45) must also be very near its maximum temperature, so that we may infer that 11,000° is the maximum for mass 2·4. This leads me to revise my idea of the average mass of the giant stars observed. I have previously regarded 1·5 as the typical mass; but since a large proportion of the giants reach type A, a larger mass (say 2·5) seems indicated. I think that this change brings me more into line with the views of Seares and von Zeipel. It also accords with the results reached in § 5.

With reference to all the numerical results, it should be understood that the present investigation is of a preliminary character. The interpolation formula (9) for the relation of pressure and density in a dwarf star is essentially provisional; and no account has been taken of the increase of molecular weight due to diminishing ionisation in the outer parts of the star. Probably it is already possible to improve the accuracy of the theory in both these respects. My object at present has been to show that a somewhat radical change of view with regard to the absorption-coefficient does not lead to any obvious inconsistency with known observational data.

5. For a giant star $1-\beta$ is always determined from the equation

$$1-\beta = \cdot 00260 \mathrm{M}^2 m^4 \beta^4 \quad . \quad . \quad . \quad . \quad (20)$$

M being in terms of the sun's mass. We can then obtain L from (18), and hence the absolute magnitude.

The principal formulæ (some of which apply only to giant stars), showing the interdependence of the variables, are (r = radius of star, T_e - effective temperature)—

$$r^2 \mathrm{T}_e^4 \propto \mathrm{L} \propto \mathrm{M}(1-\beta)/k$$

$$k \propto \rho_c / m \mathrm{T}_c^{\frac{7}{2}}$$

$$\mathrm{T}_c \propto m\beta \mathrm{M}/r$$

$$\rho_c \propto \mathrm{M}/r^3$$

From these, together with (20), we find

$$\mathrm{L} \propto \mathrm{M}^{\frac{7}{2}} (1-\beta)^{\frac{3}{2}} m^{\frac{1}{2}} \mathrm{T}_e^{\frac{4}{7}} \quad . \quad . \quad . \quad . \quad (21)$$

or

$$\mathrm{L} \propto \mathrm{M}(1-\beta)^{11/5} \beta^{-\frac{4}{5}} \mathrm{T}_e \quad . \quad . \quad . \quad . \quad (22)$$

Since in former work it was supposed that $\mathrm{L} \propto \mathrm{M}(1-\beta)$, we see from (21) that the change of magnitude with mass will be nearly $1\frac{1}{2}$ times that formerly found, apart from any corrections due to change of m.

The results of calculations made by these formulæ are given in the following table for two values of m; the magnitudes, obtained by Seares

from a discussion of the observational evidence, being added for comparison.

TABLE VI.

Absolute Bolometric Magnitudes of Giant Stars.

Mass ($\odot$=1).	m=2·8.	m=4·0.	Seares.
1	+3·8	+2·2	+3·6
1·5	+2·4	+1·1	+1·8
2	+1·4	+0·4	+0·8
3	+0·3	−0·6	−0·3
4	−0·3	−1·2	−1·0
6	−1·3	−2·0	−2·0
8	−2·0	−2·6	−2·7
10	−2·4	−3·0	−3·3
20	−3·8	−4·3	
40	−5·0	−5·5	
80	−6·2	−6·6	

Above values for type G. For other types apply the following corrections: A, −0·5; F, −0·3; K, +0·3; M, +0·6.

Seares's values have been read from his curves in *Astrophysical Journal*, **55**, p. 510. Since the curves contain an unexplained wave, they have been read at type G5, which falls about midway between crests and hollows. He has derived his masses mainly from the assumption of the equipartition of translatory energy among the stars—a somewhat speculative hypothesis, but the fair accordance with my results obtained from entirely different considerations may be regarded as lending some support to it. We should probably anticipate a molecular weight about 3·3 for the lowest masses in the table, increasing somewhat for the higher masses which have lower central temperature. The accordance with Seares's values (at the part of his curves considered) is thus as close as could be desired.

The term $T_e^{\frac{1}{2}}$ indicates an increase of luminosity of about $1^m{\cdot}1$ between type M and type A, if the mass is fixed. But against this must be set off a decrease due to lower molecular weight at the higher temperatures. It is therefore impossible to predict whether the net result is an increase or decrease. The change should in any case be small.

The zero-point of Table VI. depends on the adopted target-radius $d+\frac{1}{2}Nb=2{\cdot}0\,.\,10^{-12}$. If this were doubled or halved, all the magnitudes would be increased or decreased by $0^m{\cdot}75$. The magnitudes might thus be systematically in error by $\pm 1^m$; differentially they ought to be much more accurate. The adopted absolute magnitude of the sun is $+5^m{\cdot}1$.

Postscript.—I had overlooked the fact that we have now for one giant star, Capella, exact knowledge of the mass and absolute luminosity. According to Merrill's data (*Ap. J.*, **56**, 40), the absolute magnitude (taking the mean of the two components) is $-0^m{\cdot}1$ corresponding to mass 3·8. This satisfactorily confirms the zero of Table VI.

Summary.

The theoretical formula for the absorption-coefficient found in my last paper is here applied to determine the luminosities, effective temperatures, etc., of giant and dwarf stars of various masses.

In most of the formulæ the molecular weight m occurs raised to a rather high power, so that it is not possible to make close predictions without a very accurate knowledge of m. This depends on the ionisation, which is a function of the temperature and density not very accurately known at present. Using values of m which we think might reasonably be anticipated, the theory and the observations are quite accordant.

The new results differ from my former results chiefly in the following respects:—

(1) The maximum density to which stellar material can be compressed is considerably higher. I here adopt $\rho_0 = 12{\cdot}8$, but it is impossible to determine it with any accuracy until the ionisation is better determined.

(2) The sun has not yet dropped much below its maximum effective temperature, which was probably about 6600°.

(3) The increase of absolute magnitude for given increase of mass in giant stars is about $1\frac{1}{2}$ times that formerly adopted.